NASA-CR-65669

SCALAR AND VECTOR HELMHOLTZ INTEGRALS IN SCATTERING OF PLANE WAVES BY ONE-DIMENSIONAL ROUGH SURFACES*

N 67 34303

BY

H. S. Hayre and C. Kopala

Abstract

Scalar and vector forms of Helmholtz integral formulations of scattering of plane waves by one-dimensional rough surfaces are presented and compared. The results show that scalar form yields exact results for horizontally (perpendicular) polarized incident wave, whereas it does not seem to be adequate for vertically (parallel) polarized incident wave with field component normal to the local surface.

*This work is supported by the National Aeronautics and Space Administration under Contract No. NAS-9-6760.

Introduction

Current theories of scattering of electromagnetic energy by rough surfaces are based upon solutions of the scalar integral

$$E_2 = \frac{1}{4\pi}\int_S \left\{ (E)_s \frac{\partial\phi}{\partial n} - \phi \left(\frac{\partial E}{\partial n}\right) \right\} ds \tag{1}$$

with the boundary conditions obtained by Beckmann (1963),

$$(E)_s = (1 + R)E_1 \tag{2}$$

$$\left(\frac{\partial E}{\partial n}\right)_s = (1 - R) \tag{3}$$

where ϕ is the free space Green's function

$$\phi = \frac{1}{r} \exp\left\{ i\ \bar{K}_2 \cdot \bar{r} \right\} \tag{4}$$

The propagation vector of the incident energy is $\bar{K}_1$ and of the scattered energy is $\bar{K}_2$. The normal to the surface is $\bar{n}$. The reflection coefficient R is that for a perfectly conducting plane surface and takes the value +1 in the case of vertically polarized incident radiation. In the case of horizontally polarized incident radiation it has the value of -1.

For the one dimensional surface illustrated in Figure 1, the general expression for the scattered field obtained by Beckmann (1963) is

$$E_2 = \frac{i \exp\left\{ iKR_0 \right\}}{4\pi R_0} \int_{-L}^{L} (a\zeta' - b) \exp\left\{ i(v_x x + v_z \zeta) \right\} dx \tag{5}$$

where

$$a = (1-R) \sin \theta_1 + (1+R) \sin \theta_2 \tag{6}$$

$$b = (1+R) \cos \theta_2 - (1-R) \cos \theta_1 \tag{7}$$

$$v_x = K (\sin \theta_1 - \sin \theta_2) \tag{8}$$

$$v_z = -K (\cos \theta_1 + \cos \theta_2) \tag{9}$$

and R_o is the distance from the origin to the point of observation.

A vector formulation of the theory of scattering of electromagnetic energy by rough surfaces has been developed, and, as is to be expected, is not in complete agreement with the scalar formulation. This paper presents the vector formulation, compares the vector and scalar approaches and discusses the significant differences.

The Vector Formulation

Evaluation of the integral developed by Stratton and Chu (1941)

$$\bar{E}_2 = -\frac{1}{4\pi} \iint_S \left\{ i\omega\mu\phi\bar{n} \times \bar{H} + (\bar{n} \cdot \bar{E}) \nabla\phi + (\bar{n} \times \bar{E}) \times \nabla\phi \right\} ds \tag{10}$$

will yield an expression for the scattered electric field. The surface S is assumed to posses the following restrictive properties:

1. The surface is continuous
2. Mutual interactions between the irregularities of the surface may be neglected.
3. Shadowing does not occur

4. The field on the surface may be approximated by that on a perfectly conducting plane surface of infinite extent tangent to the surface.

The surface S is defined by the radius vector

$$\bar{r}_s = x\ \bar{a}_x + \zeta(x)\ \bar{a}_z \tag{11}$$

where $\zeta(x)$ is the height of the surface above the plane Z = 0 at X. The boundary conditions are

$$(\bar{n} \times \bar{E}_1)_s = 0 \tag{12}$$

$$(\bar{n} \cdot \bar{E})_s = 2\ \bar{n}_1 \bar{E}_1 \Big|_{\bar{r}=\bar{r}_s} \tag{13}$$

$$(\bar{n} \times \bar{H})_s = 2\ \bar{n} \times \bar{H}_1 \Big|_{\bar{r}=\bar{r}_s} \tag{14}$$

where $\bar{E}_1$ is the incident electric field and $\bar{H}_1$ is the magnetic field corresponding with $\bar{E}_1$. The Green's function ϕ is the free space Green's function

$$\phi = \frac{1}{r} \exp \left\{ i\bar{K}_2 \cdot \bar{r} \right\} \tag{15}$$

where the propagation vector $\bar{K}_2$ is

$$\bar{K}_2 = K_2 \left\{ \sin\ \theta_2\ \bar{a}_x + \cos\ \theta_2\ \bar{a}_z \right\} \tag{16}$$

For vertically polarized incident energy, the electric field is taken to be

$$\bar{E}_1 = \left\{ E_x\ \bar{a}_x + E_z\ \bar{a}_z \right\} \tag{17}$$

and the magnetic field is

$$\bar{H}_1 = \frac{i\bar{a}_y}{c} \left\{ E_z\ \sin\ \theta_1 + E_x\ \cos\ \theta_1 \right\} \exp \left\{ i(\bar{K}_1 \cdot \bar{r} - \omega t) \right\} \tag{18}$$

where C is the velocity of light. For horizontally polarized incident energy, the incident electric field is

$$\bar{E}_1 = \bar{E}_y \, \bar{a}_y \exp\left\{ i(\bar{K}_1 \cdot \bar{r} - \omega t) \right\} \tag{19}$$

and the magnetic field is

$$\bar{H}_1 = \frac{E_y K_1}{i\omega\mu} \left\{ \bar{a}_x \cos \theta_1 - \bar{a}_z \sin \theta_1 \right\} \exp\left\{ i\bar{K}_1 \cdot \bar{r} \right\} \tag{20}$$

Evaluation of (10) for the case of vertically polarized incident energy yields

$$E_2 = - \frac{\exp\left\{ iK_2 R_o \right\}}{2\pi \, R_o} \int_{-L}^{L} \left[\left\{ \alpha + \beta S' + \gamma [S']^2 \right\} \bar{a}_x \left\{ \epsilon + \xi S' \right\} \right] \exp\left\{ i(v_x x + v_x S) \right\} \tag{21}$$

where

$$\alpha = -iK_2 E_z \sin \theta_2 + K_1 (E_z \sin \theta + E_x \cos \theta_1) \tag{22}$$

$$\beta = iK_2 (E_x \sin \theta_2 - E_z \cos \theta_2) \tag{23}$$

$$\gamma = iK_2 \, E_x \cos \theta_2 \tag{24}$$

$$\epsilon = iK_2 \, E_z \cos \theta_2 \tag{25}$$

$$\xi = iK_2 \, E_x \cos \theta_2 + K_1 \, (E_z \sin \theta_1 + E_x \cos \theta_1) \tag{26}$$

$$v_x = K_1 \sin \theta_1 - K_2 \sin \theta_2 \tag{27}$$

$$v_z = -(K_1 \cos \theta_1 + K_2 \cos \theta_2) \tag{28}$$

Evaluation of (10) for the case of horizontally polarized incident radiation results in the expression

$$E_2 = \frac{a_y E_y K_1 \exp\left\{ iK_2 R_o \right\}}{2\pi R_o} \int_{-L}^{L} \left\{ \cos \theta_1 + \sin \theta_1 S' \right\} \exp\left\{ i(v_x x + v_z S) \right\} dx \tag{29}$$

where v_x and v_z are given by (27) and 28), respectively.

Analysis

Evaluation of (5) for the case of horizontally polarized incident energy results in

$$E_2 = \frac{i \exp\{iKR_o\}}{4\pi R_o} \int_{-L}^{L} (2 \sin \theta_1 S' + 2 \cos \theta_1) \exp\left\{i(v_x x + v_z S)\right\} dx \quad (30)$$

which is identical with (29). Evaluation of (5) for the case of vertically polarized incident energy results in

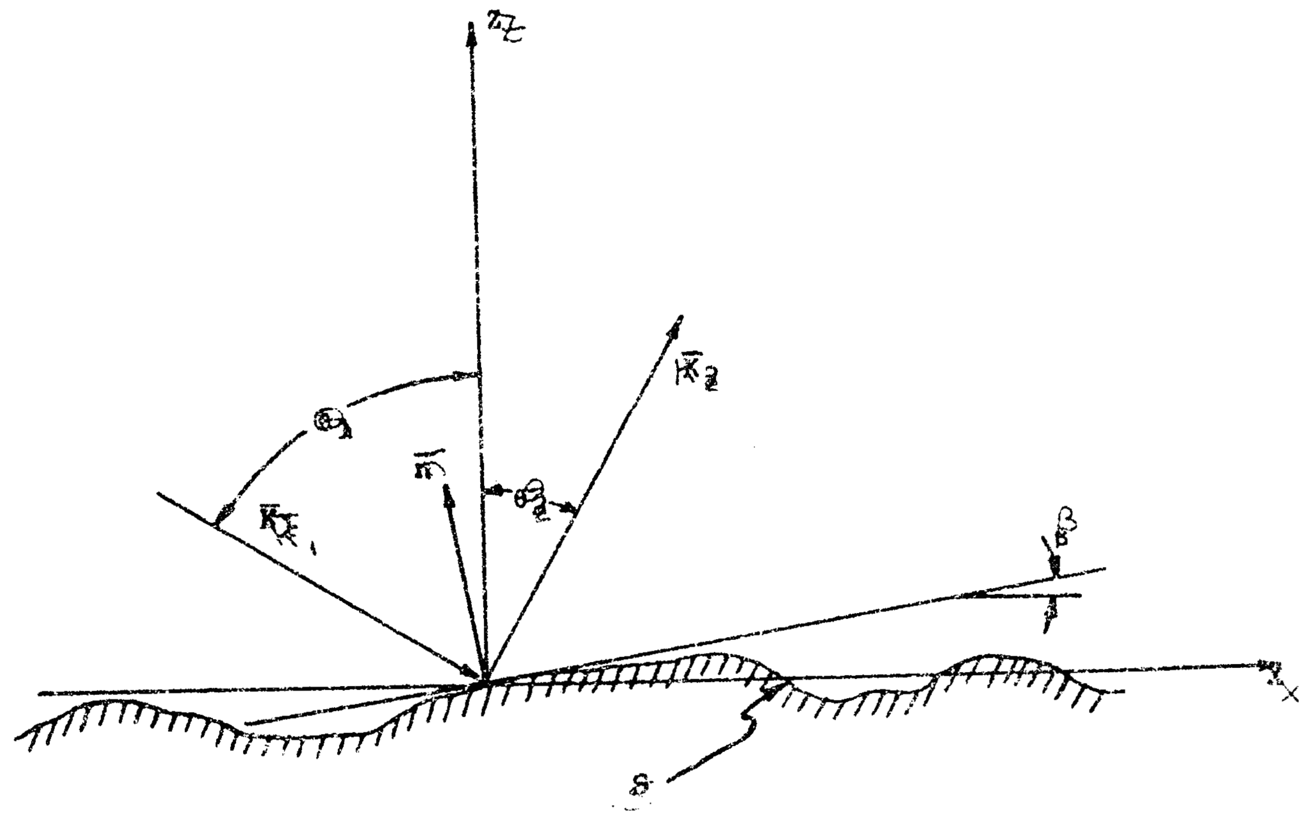
$$E_2 = \frac{i \exp\{iKR_o\}}{4\pi R_o} \int_{-L}^{L} (2 \sin \theta_2 S' - 2 \cos \theta_2) \exp\left\{i(v_x x + v_z S)\right\} dx \quad (31)$$

which, when compared with (21) shows few similarities.

The polarization is coincidental since the above results show that the boundary conditions can be simply expressed in relation to the surface in case of the horizontal polarization and the vector field to be treated as three scalars.

Conclusion

The case of horizontally polarized plane wave scattering from a one dimensional surface can be studied by using scalar Helmholtz integral approach whereas it is not the case for vertical polarization. Extension of this work to two dimensional surfaces, which is in progress, seems to indicate that the analogy may fail for both polarizations.



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The Geometry of a One-Dimensional Rough Surface.

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